CHAPTER 13

1

Conducting Probability Experiments

Goal **Compare probabilities in two experiments.**

Game 1

1. Place a shuffled deck of cards face down.
2. Turn over the top card.
3. If the card is an ace, you get 4 points.

A player wins if he or she has at least 10 points after 4 turns.

Game 2

1. Place a shuffled deck of cards face down.
2. Turn over the top card.
3. If the card is a red card (heart or diamond), you get 2 points.

A player wins if she or he has at least 6 points after 4 turns.

At-Home Help

Probability refers to the likelihood that an event will happen.

For example, if you flip a coin, there are two possible outcomes. You can get either heads or tails.

The probability of getting heads is the same as the probability of getting tails.

1. Predict which game you are more likely to win. Justify your prediction.

Suggested answer: I think I'm more likely to get a red card than an ace. There are only 4 aces in the deck, but there are 26 red cards. I think I'm more likely to win Game 2.

2. Tammy played both games three times.
Which game are you more likely to win? Use probability language to explain why.

Game 1					Points
Turn number Ace?	1 ×	2 ×	3 ×	4 ✓	4
Turn number Ace?	1 ×	2 ×	3 ✓	4 ×	4
Turn number Ace?	1 ×	2 ×	3 ×	4 ×	0

Game 2					Points
Turn number Red card?	1 ✓	2 ✓	3 ×	4 ✓	6
Turn number Red card?	1 ×	2 ✓	3 ✓	4 ✓	6
Turn number Red card?	1 ✓	2 ×	3 ✓	4 ✓	6

Suggested answer: You're more likely to win Game 2. Tammy won Game 2 three times but lost Game 1 every time. For Tammy to win Game 1, she would have to turn over 3 aces. Since there are only 4 aces in the deck of 52, getting 3 of them would be very unlikely.

For Tammy to win Game 2, she would have to turn over 3 red cards. Since there are 26 red cards in the deck of 52, it is more likely that Tammy can flip 3 of them to win.

Using Percents to Describe Probabilities

Goal **Conduct experiments and use percent to describe probabilities.**

At-Home Help

Probabilities can be written as percents.

For example, if you rolled a die 10 times and got a 4 three times, the probability of rolling a 4 would be 3 out of 10, or $\frac{3}{10}$.

To express $\frac{3}{10}$ as a percent, find an equivalent fraction.

$\frac{3}{10} = \frac{30}{100}$

$= 30\%$

1. Siegfried rolled a die 20 times.

Roll	1	2	3	4	5	6	7	8	9	10
Number on die	2	3	1	4	2	6	3	5	2	1
Roll	11	12	13	14	15	16	17	18	19	20
Number on die	5	3	1	2	6	4	1	3	2	4

Record the probability of each event as a percent.

a) rolling a 1

$\frac{4}{20} = \frac{20}{100}$

$= 20\%$

b) rolling a multiple of 3

$\frac{6}{20} = \frac{30}{100}$

$= 30\%$

c) rolling an odd number

$\frac{10}{20} = \frac{50}{100}$

$= 50\%$

d) rolling a 7

$\frac{0}{20} = \frac{0}{100}$

$= 0\%$

2. The probability of winning a game is 40%. Predict how many times you expect to win in each situation.

a) if you play 10 times

$\frac{40}{100} = \frac{4}{10}$

4 times

b) if you play 25 times

$\frac{40}{100} = \frac{10}{25}$

10 times

c) if you play 50 times

$\frac{40}{100} = \frac{20}{50}$

20 times

3. a) Roll a die 25 times and record each roll.

Suggested answer:

Roll	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Number on die	2	1	3	4	2	5	1	6	3	2	1	3	4	5	2	1	6	4	2	3	5	4	1	6	2

b) Record each probability as a percent.

i) rolling an even number

$\frac{13}{25} = \frac{52}{100}$

$= 52\%$

ii) rolling a number less than 5

$\frac{19}{25} = \frac{76}{100}$

$= 76\%$

iii) rolling a number less than 10

$\frac{25}{25} = \frac{100}{100}$

$= 100\%$

CHAPTER 13

3

Solving a Problem by Conducting an Experiment

Goal **Use an experiment as a problem solving strategy.**

Jessica and her brother use a die to decide who will help with dinner each night. An even number means it is Jessica's turn. An odd number means it is her brother's turn. Conduct an experiment to determine the probability that Jessica will help with dinner more than 3 times in the next week.

At-Home Help

To determine the probability in a problem, conduct an experiment.

All of the events in the experiment should be random. A result is random if what happens is based on chance. Something that is not random has to happen a certain way.

For example, the day after Tuesday is always Wednesday. That is not random. If you put the names of the days of the week in a bag and pick one name, the result is random.

Remember to conduct many experiments before determining the probability in the problem.

Suggested answer:

Understand the Problem
I need to calculate the fraction of the days that Jessica will help with dinner in a week.

Make a Plan
I'll conduct an experiment. I'll roll a die 7 times. Each roll represents a day of the week. I'll record the results of each roll in a chart.

I'll do the experiment 20 times and see what fraction of the days Jessica will help with dinner. If I get more than 3 even numbers in an experiment, Jessica will help with dinner more than 3 times that week.

Carry Out the Plan
These are my results.

Experiment	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Roll 1	2	6	1	6	3	4	3	5	3	2	4	4	5	1	2	6	2	1	2	5
Roll 2	1	5	1	2	1	2	1	3	2	3	3	2	4	2	4	2	1	3	4	3
Roll 3	3	2	4	1	2	3	5	2	1	1	1	3	2	3	1	3	1	2	3	4
Roll 4	5	4	3	3	1	1	6	1	1	5	3	1	1	1	3	5	3	4	3	2
Roll 5	4	2	2	1	5	2	2	4	6	5	6	4	3	5	4	3	5	3	5	3
Roll 6	6	3	2	2	5	6	3	3	4	4	5	6	4	6	5	4	6	5	6	2
Roll 7	1	1	5	4	4	5	4	1	2	1	1	5	2	1	6	1	2	1	2	4
		✓		✓		✓			✓			✓	✓		✓				✓	✓

I got more than 3 even numbers in 9 of the 20 experiments. So the probability of Jessica helping with dinner more than 3 times in a week is $\frac{9}{20}$. That is equivalent to $\frac{45}{100}$ or 45%.

Look Back
There are 3 even numbers and 3 odd numbers on a die. So if I roll a die, there is a 3 in 6 chance of getting an even number. If I roll a die 7 times, I expect to get an even number either 3 or 4 times. My result of 45% looks reasonable

CHAPTER 13

4

Theoretical Probability

Goal **Create a list of all possible outcomes to determine a probability.**

At-Home Help

Theoretical probability is the probability you would expect when you analyze all of the different possible outcomes.

For example, the theoretical probability of flipping a head on a coin is $\frac{1}{2}$, since there are 2 equally likely outcomes and only 1 is favourable.

Experimental probability is the probability that actually happens when you do the experiment.

1. If you shuffle a deck of cards, what is the theoretical probability of each event?

a) picking an ace

$\frac{4}{52}$ or $\frac{1}{13}$

b) picking a spade

$\frac{13}{52}$ or $\frac{1}{4}$

c) picking a face card

$\frac{12}{52}$ or $\frac{3}{13}$

2. If you roll a die two times, what is the theoretical probability of each event?

a) sum of 6 $\frac{5}{36}$

b) sum of 10 $\frac{3}{36}$ or $\frac{1}{12}$

Roll 1

		1	2	3	4	5	6
Roll 2	1	2	3	4	5	⑥	7
	2	3	4	5	⑥	7	8
	3	4	5	⑥	7	8	9
	4	5	⑥	7	8	9	10
	5	⑥	7	8	9	10	11
	6	7	8	9	10	11	12

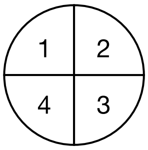
c) difference of 5 $\frac{2}{36}$ or $\frac{1}{18}$

d) difference of 2 $\frac{8}{36}$ or $\frac{2}{9}$

Roll 1

		1	2	3	4	5	6
Roll 2	1	0	1	2	3	4	⑤
	2	1	0	1	2	3	4
	3	2	1	0	1	2	3
	4	3	2	1	0	1	2
	5	4	3	2	1	0	1
	6	⑤	4	3	2	1	0

3. Imagine spinning this spinner twice.



a) What is the theoretical probability that the sum of the two spins is greater than 4? $\frac{10}{16}$ or $\frac{5}{8}$

b) What is the theoretical probability that the sum is an odd number? $\frac{8}{16}$ or $\frac{1}{2}$

Spin 1

		1	2	3	4
Spin 2	1	2	3	4	⑤
	2	3	4	⑤	⑥
	3	4	⑤	⑥	⑦
	4	⑤	⑥	⑦	⑧

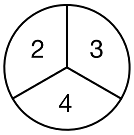
CHAPTER 13

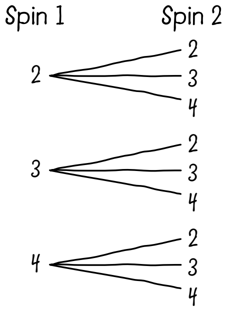
5

Tree Diagrams

Goal **Use a tree diagram to determine a theoretical probability.**

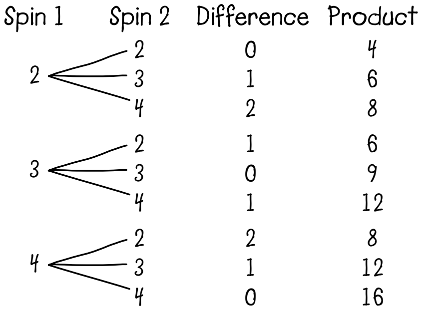
1. **a)** Use a tree diagram to list the possible outcomes if this spinner is spun twice.





b) Determine the theoretical probability that the difference of the numbers is 0.

Tree diagram for parts b) and c):



$\frac{3}{9}$ or $\frac{1}{3}$

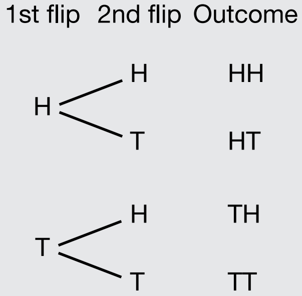
c) Determine the theoretical probability that the product of the numbers is greater than 6.

$\frac{6}{9}$ or $\frac{2}{3}$

At-Home Help

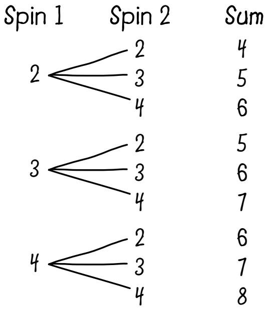
A **tree diagram** is a way to record and count all combinations of events, using lines to form branches.

For example, the tree diagram below shows all the possible outcomes if you flip a coin twice.



2. Nathan and Jay are playing a game with the spinner in Question 1. Nathan wins if his two spins give a sum greater than 5. Otherwise, Jay wins. Use a tree diagram to explain if this game is fair.

Suggested answer: The theoretical probability of getting a sum greater than 5 is $\frac{6}{9}$ or $\frac{2}{3}$. A game is fair if each player has an equal chance of winning. If there are two players, each player should have a 50% chance of winning. In this game, Nathan is more likely to get a sum greater than 5. So the game is not fair.



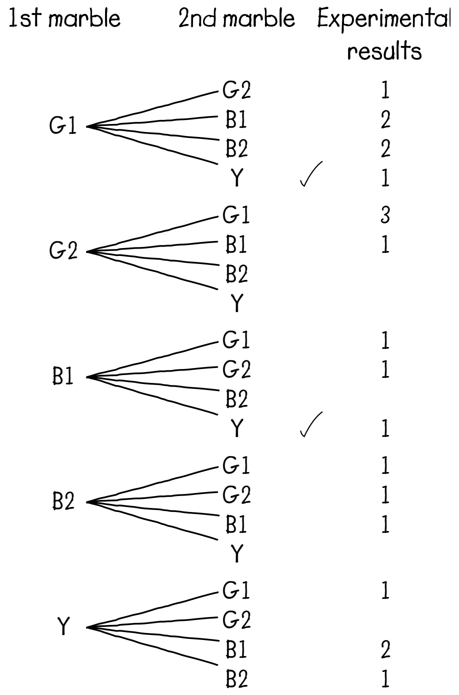
6 Comparing Theoretical and Experimental Probability

Goal **Compare the theoretical probability of an event with the results of an experiment.**

1. Two green marbles, two blue marbles, and one yellow marble are placed in a bag. The marbles are mixed up and two marbles are picked, one at a time, without looking.

a) What is the theoretical probability of picking a green marble and then a yellow one? Use a tree diagram.

Tree diagram for parts a) and b):



$\frac{2}{20}$ or $\frac{1}{10}$

b) Conduct an experiment 20 times. What is your experimental probability for this event? Record your results beside your tree diagram in part **a)**.

Suggested answer: $\frac{1}{20}$

c) Why might the experimental probability be different from the theoretical probability?

Suggested answer: The experimental probability was different because I only did the experiment 20 times. Also, I might not have mixed up the marbles well enough between experiments.

At-Home Help

To determine the theoretical probability of an event, you can use a tree diagram to list all possible outcomes.

To determine the experimental probability of that event, conduct an experiment.

Before comparing theoretical and experimental probabilities, make sure the experiment was conducted many times.

Usually experimental probabilities are not the same as theoretical probabilities. If you do a great enough number of experiments, the experimental probability will be the same as or very close to the theoretical one.

Test Yourself Page 1

Circle the correct answer.

Use the chart to answer Questions 1 and 2.

Nazir's Rolls of a Die

First 5 rolls	2	1	6	3	5
Next 5 rolls	1	1	4	2	5
Next 5 rolls	3	2	4	5	1

1. What is the probability of Nazir rolling an even number in the first 10 rolls?

A. $\frac{3}{10}$ [✓ **B.** $\frac{4}{10}$] **C.** $\frac{7}{10}$ **D.** $\frac{3}{5}$

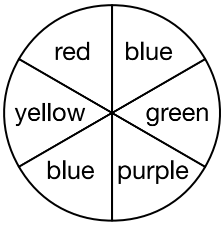
2. What is the probability of Nazir rolling a number greater than 4 in all 15 rolls?

[✓ **A.** $\frac{4}{15}$] **B.** $\frac{1}{3}$ **C.** $\frac{7}{15}$ **D.** $\frac{11}{15}$

3. What is the theoretical probability of spinning blue on this spinner?

A. $\frac{1}{6}$ **C.** $\frac{2}{3}$

[✓ **B.** $\frac{2}{6}$] **D.** $\frac{1}{2}$



4. Renata spun the spinner in Question 3 10 times. What is the probability of Renata spinning blue?

Spin number	1	2	3	4	5	6	7	8	9	10
Colour	blue	yellow	green	red	green	green	purple	blue	blue	red

A. 10% **B.** 20% [✓ **C.** 30%] **D.** 40%

5. What is the theoretical probability of flipping a coin three times and getting heads all three times?

[✓ **A.** $\frac{1}{8}$] **B.** $\frac{1}{4}$ **C.** $\frac{3}{8}$ **D.** $\frac{1}{2}$

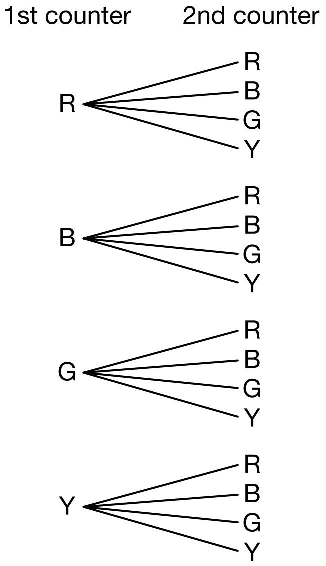
6. What is the theoretical probability of picking an ace from a shuffled deck of cards?

A. $\frac{1}{52}$ [✓ **B.** $\frac{4}{52}$] **C.** $\frac{1}{2}$ **D.** $\frac{3}{4}$

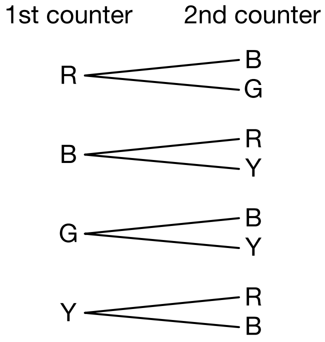
Test Yourself Page 2

7. One red counter, one blue counter, one green counter, and one yellow counter are placed in a bag. The counters are mixed up and two counters are picked, one at a time, without looking. Each time a counter is picked, it is not replaced in the bag. Which tree diagram represents all possible outcomes?

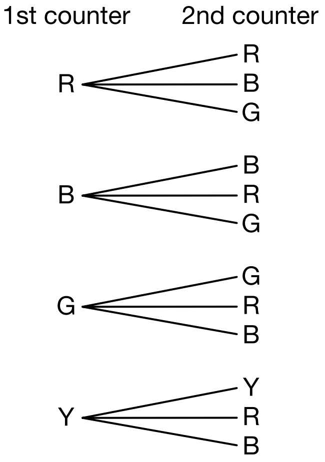
A.



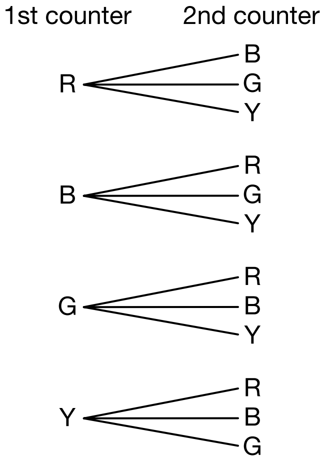
C.



B.



[✓ **D.**]



8. What is the theoretical probability of picking a green counter and a yellow counter (in any order) for the situation in Question 7?

A. $\frac{1}{12}$ [✓ **B.** $\frac{2}{12}$] **C.** $\frac{1}{4}$ **D.** $\frac{1}{3}$